De-noising on the Body Centered Cubic (BCC) Sampling Lattice

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Motivation

- Why is BCC superior to Cartesian (CC) in medical imaging?
  - 3D: saves 30% samples
  - 3D time-varying: saves 50% samples
  - Higher dimensions: potentially higher savings
- BCC grid seems well-positioned to take over the CC grid in medical imaging

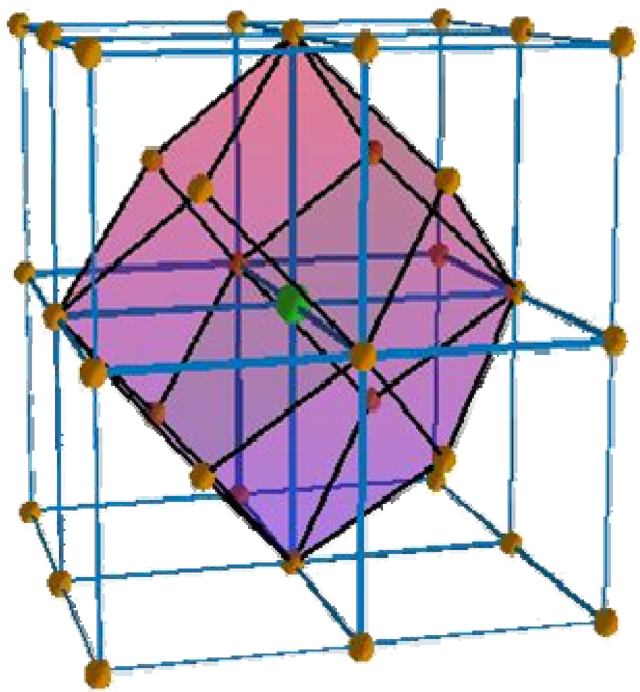
Motivation

- Why is BCC not used in medical imaging?
  - Few tools for the BCC grid exist
- Why de-noising on the BCC lattice?
  - De-noising is necessary in medical imaging
  - De-noising tools for BCC does not exist
- Ideal goal: BCC is no worse than CC in de-noising

Abstract

- Two types of noise investigated
  - Salt & pepper noise: impulse noise
  - Gaussian white noise: random noise
- Two filters investigated
  - Median filter: salt & pepper noise
  - Gaussian smoothing filter: white noise
- Error plots of CC vs BCC de-noising

The BCC Lattice



- Start with canonical CC lattice
- A lattice point belongs to the BCC lattice if and only if all three of its coordinates are even, or if all three are odd

Sampling Equivalence

- Theorem
  - Consider an unknown 3D signal. On average, to capture the same amount of information via sampling, it takes the BCC lattice roughly 70% of the number of samples that it would take the CC lattice
  - In the limit, the exact percentage is 1/sqrt(2) ~= 70.7%

Stage 1: Sampler

- Marschner Lobb (ML) dataset
- CC: 64 x 64 x64 samples
- BCC: 45 x 45 x 90 samples
- The number of samples in BCC dataset is roughly 70% of that of CC dataset
- By theorem, they capture roughly the same amount of information from ML

Stage 1: Sampler

- CC/BCC pair: noise free
- CC/BCC pair: salt & pepper noise
  - Input: probability of noise
- CC/BCC pair: Gaussian noise
  - Input: standard deviation = average difference of noise samples

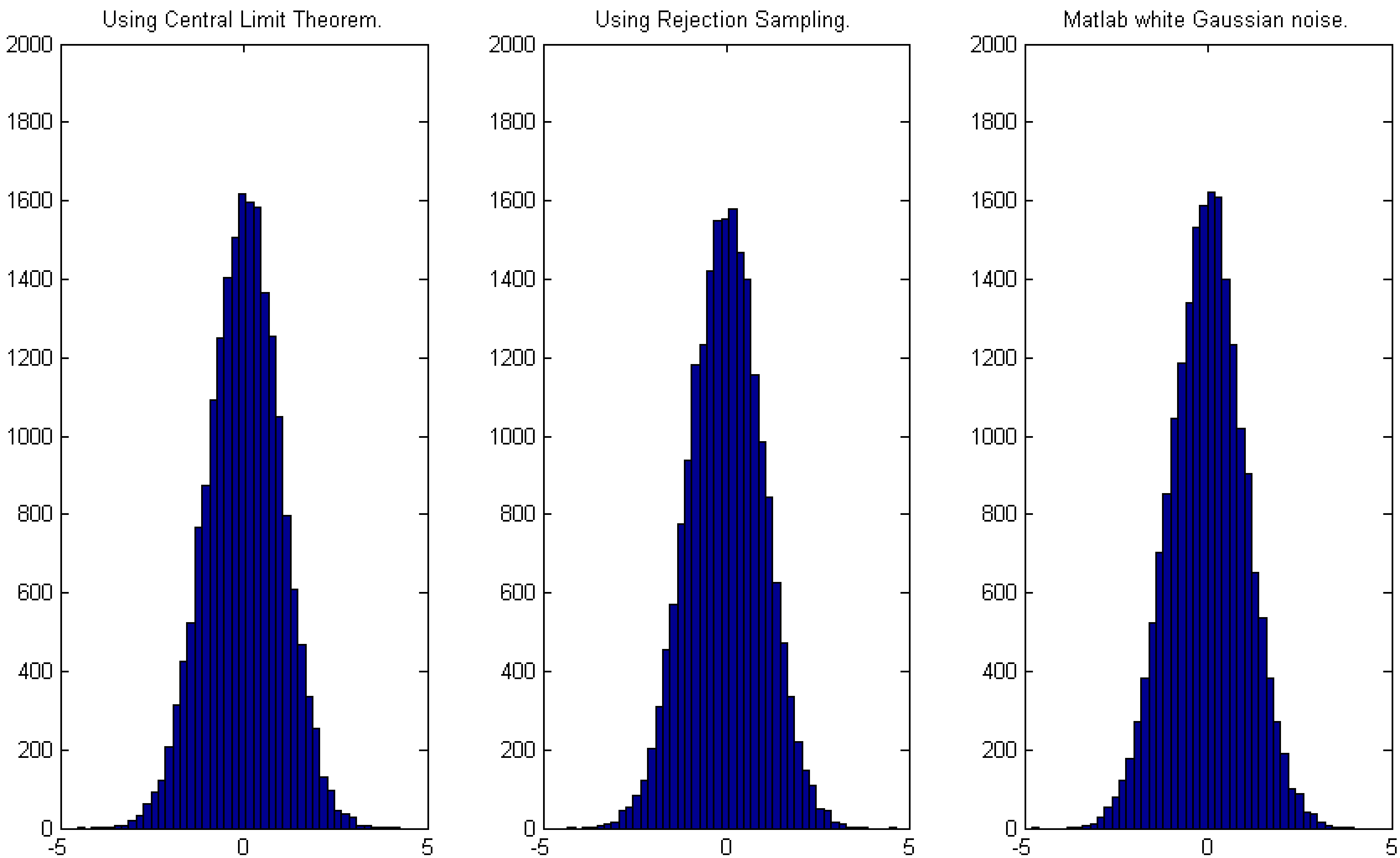
Salt & Pepper Noise Generation

- Input: probability p
- For each sample, generate y = rand[0..1]
  - If 0 <= y < p/2, set sample to 0 (pepper)
  - If p/2 <= y <= p, set sample to 254 (salt)
  - Else leave sample alone

White Noise Generation

- Method 1: The Central Limit Theorem states that the sum of N random numbers will approach normal distribution as N approaches infinity; N >= 30 works
- Method 2: Rejection sampling; dart throwing till a sample falls under the Gaussian envelope; very slow

White Noise Generation



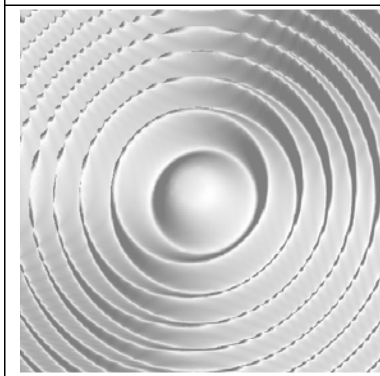
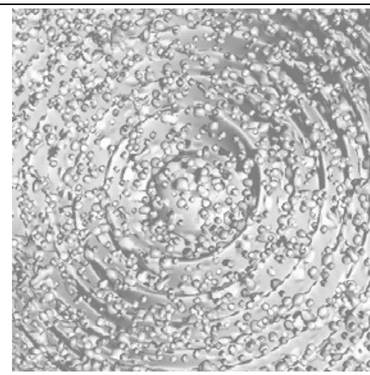
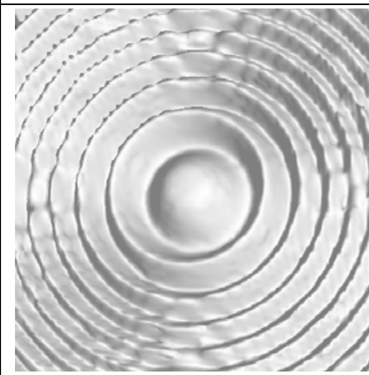
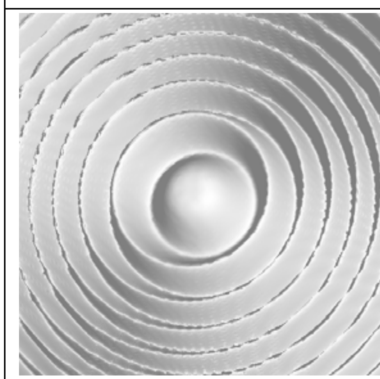
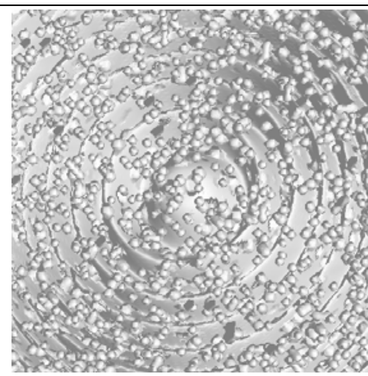
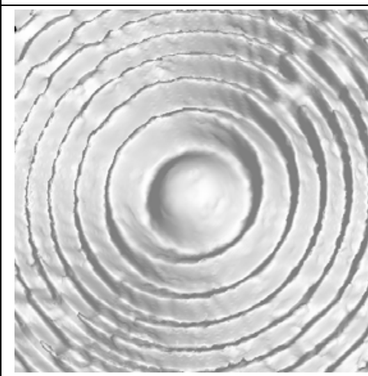
Stage 1: De-noiser

- Input: filter radius, noise type, grid type
- Noise type, grid type -> choose filter
  - Set that filter to the input filter radius
  - Apply the filter to the dataset

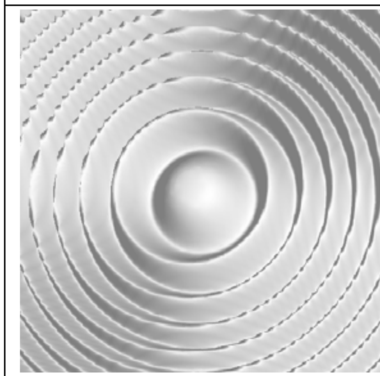
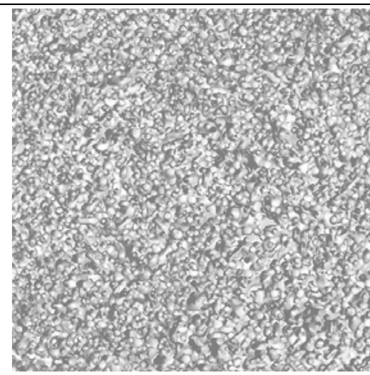
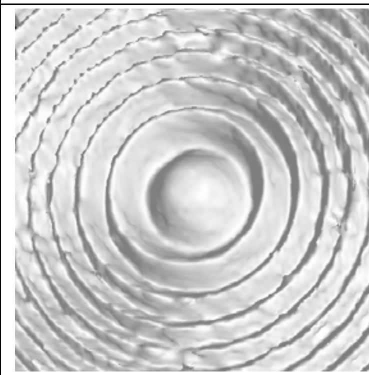
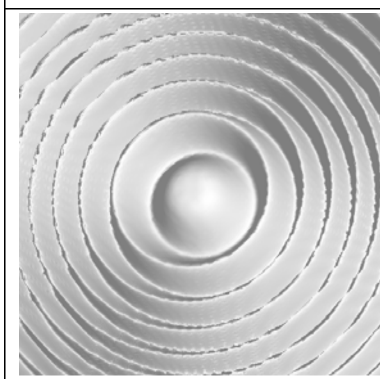
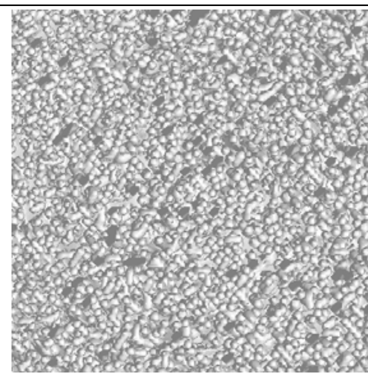
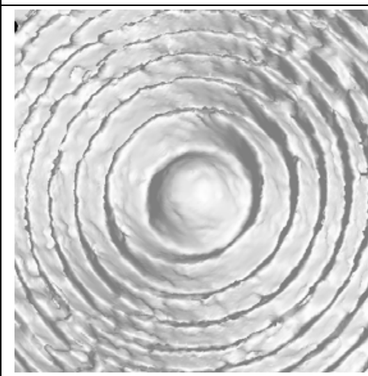
Stage 1: De-noiser

- Four filters to choose from:
  - CC median filter
  - BCC median filter
  - CC Gaussian filter
  - BCC Gaussian filter

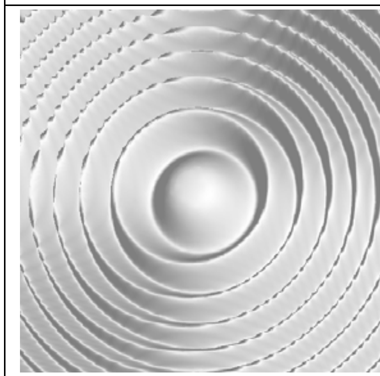
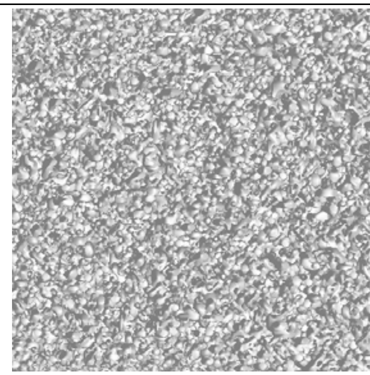
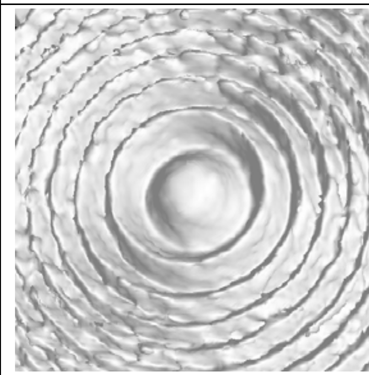
Salt & Pepper: Low Noise

CC Original	CC Salt & Pepper 3%	Filter size = 1.414214 Neighborhood size = 19
		
BCC Original	BCC Salt & Pepper 3%	Filter size = 1.415730 Neighborhood size = 15
		

Salt & Pepper: Medium Noise

CC Original	CC Salt & Pepper 10%	Filter size = 1.414214 Neighborhood size = 19
		
BCC Original	BCC Salt & Pepper 10%	Filter size = 1.415730 Neighborhood size = 15
		

Salt & Pepper: High Noise

CC Original	CC Salt & Pepper 20%	Filter size = 1.414214 Neighborhood size = 19
		
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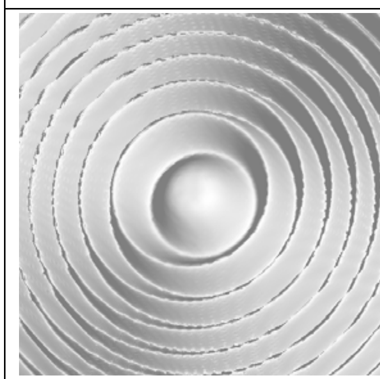
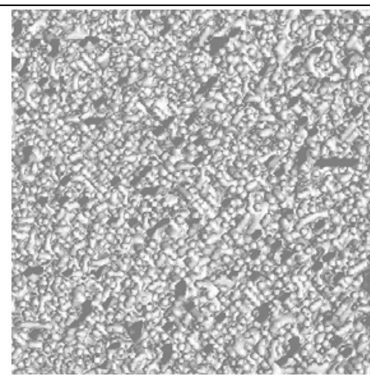
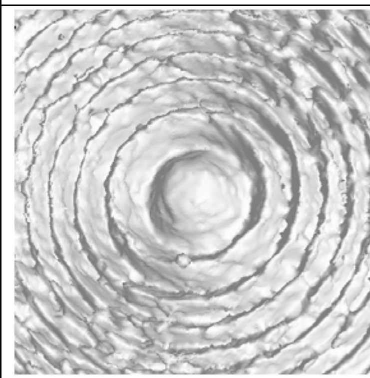
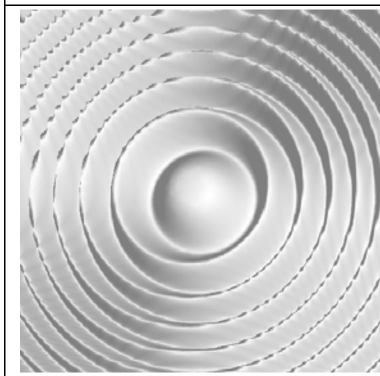
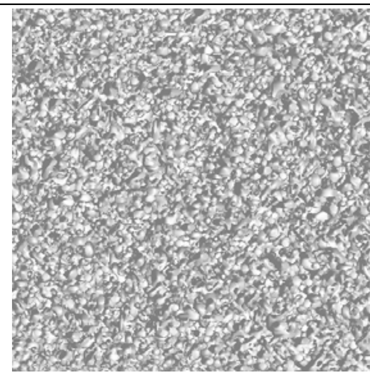
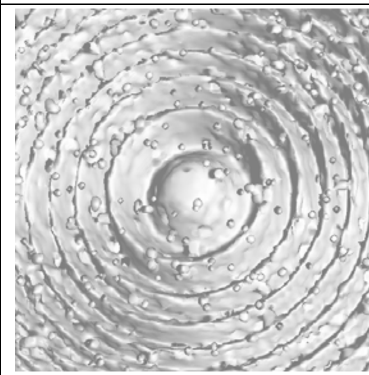
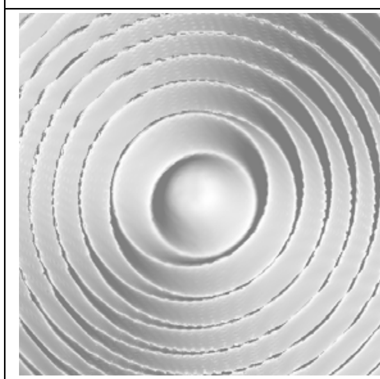
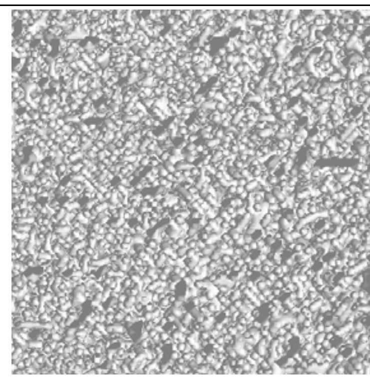
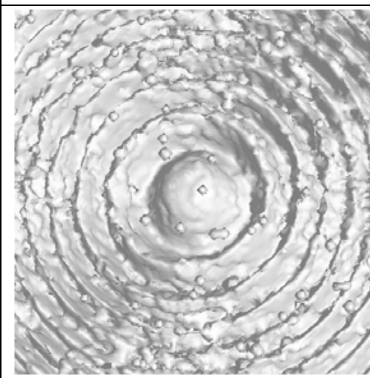
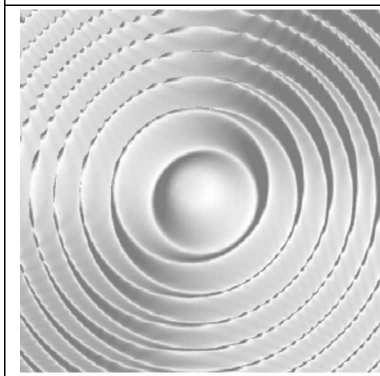
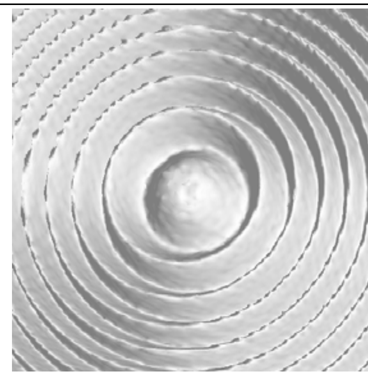
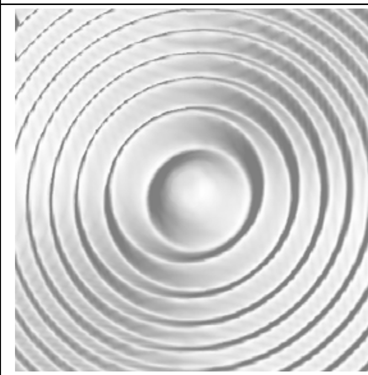
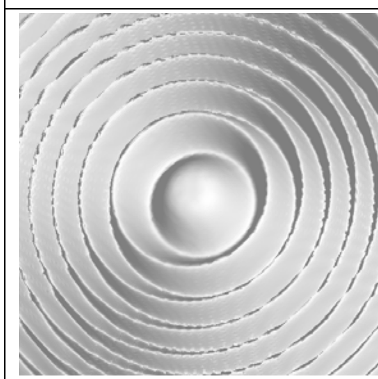
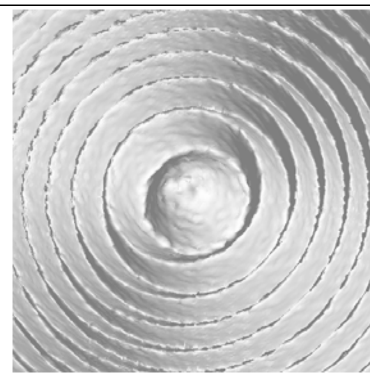
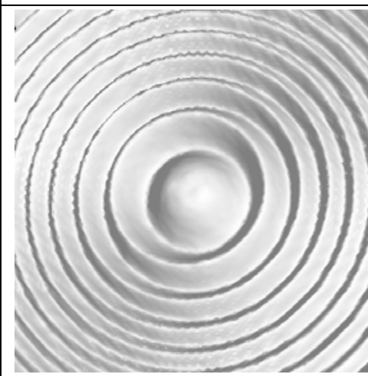
		

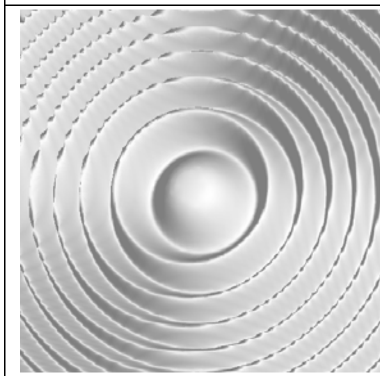
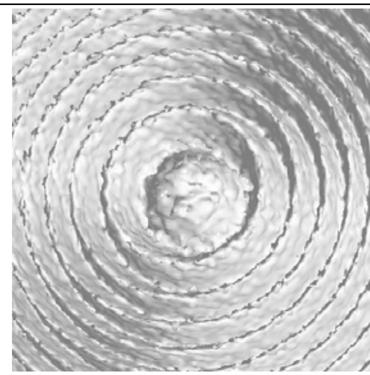
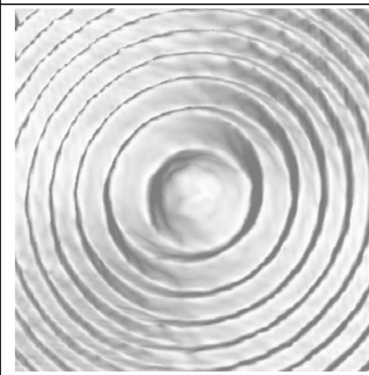
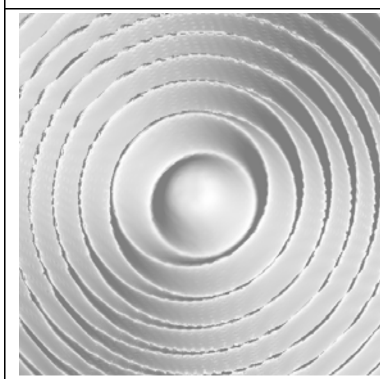
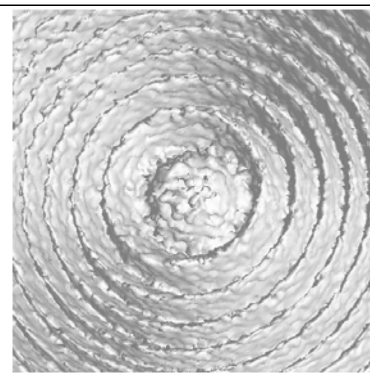
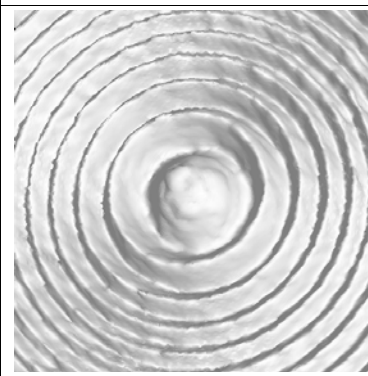
Salt & Pepper: High Noise

CC Original	CC Salt & Pepper 20%	Filter size = 1 Neighborhood size = 7
		
BCC Original	BCC Salt & Pepper 20%	Filter size = 1.226058 Neighborhood size = 9
		

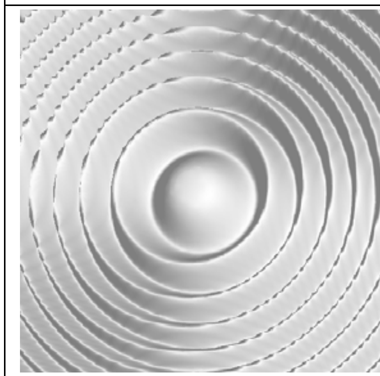
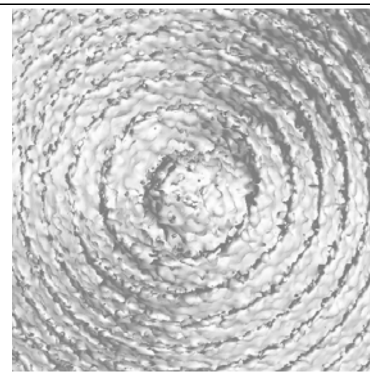
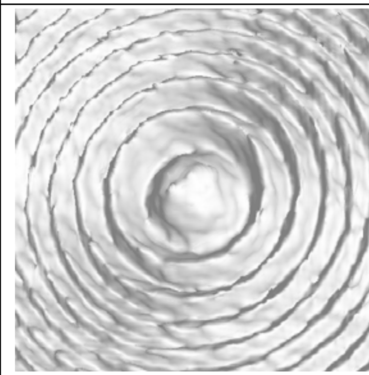
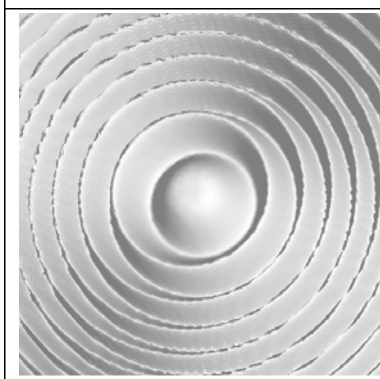
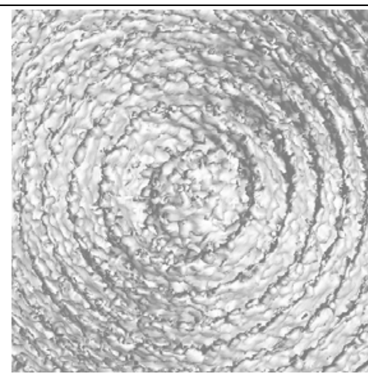
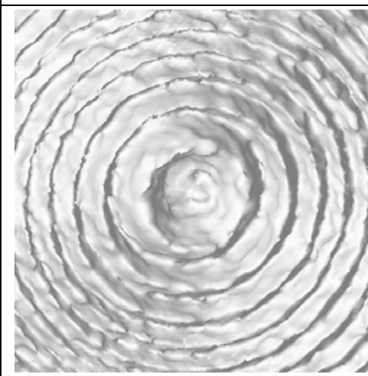
White Noise: Low Noise

CC Original	CC Gaussian Sigma = 2	Filter size = 2.236068 Neighborhood size = 57
		
BCC Original	BCC Gaussian Sigma = 2	Filter size = 2.347723 Neighborhood size = 51
		

White Noise: Medium Noise

CC Original	CC Gaussian Sigma = 7	Filter size = 2.236068 Neighborhood size = 57
		
BCC Original	BCC Gaussian Sigma = 7	Filter size = 2.347723 Neighborhood size = 51
		

White Noise: High Noise

CC Original	CC Gaussian Sigma = 14	Filter size = 2.236068 Neighborhood size = 57
		
BCC Original	BCC Gaussian Sigma = 14	Filter size = 2.347723 Neighborhood size = 51
		

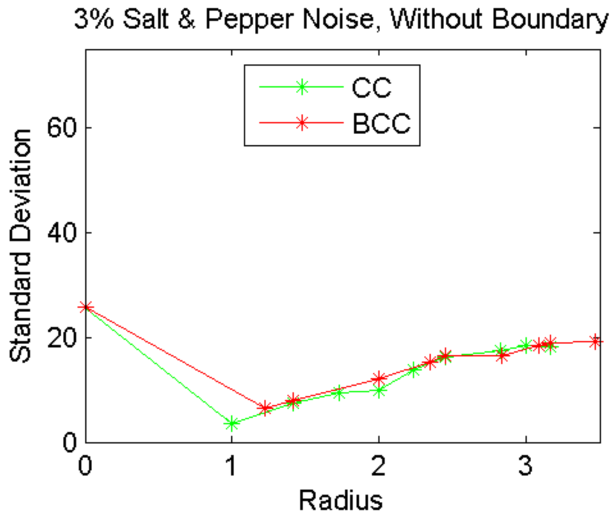
Stage 2: Data Plot

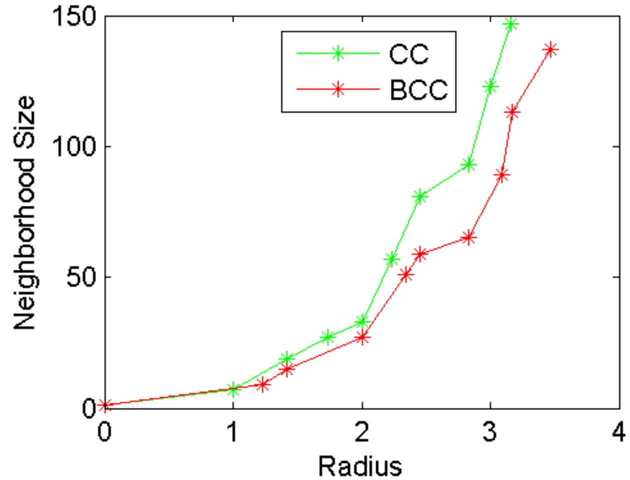
- Error metric after de-noise:
  - Compute difference between de-noised dataset and noise-free dataset
  - Mean ~= 0, so can be ignored
  - Use standard deviation as error metric

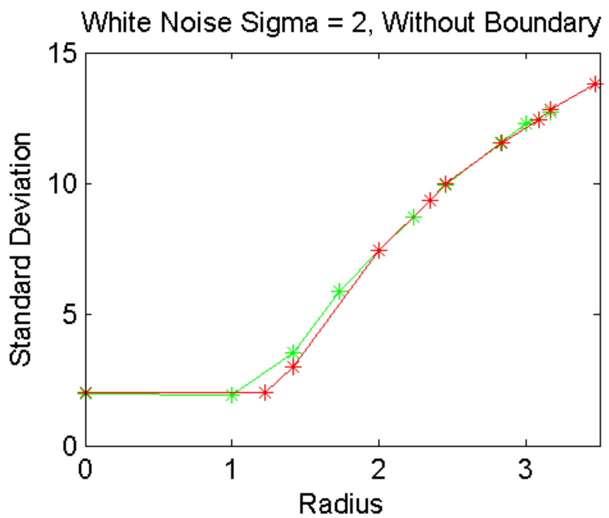
Stage 2: Data Plot

- One 2D point for each neighbourhood
  - Filter radius corresponding to neighbourhood
  - Error after filtering with this radius
- Generate 10 points for first 10 neighbourhoods
  - Can already see a convergent behaviour
  - Larger neighbourhood => slower filtering

Plot: Low Noise

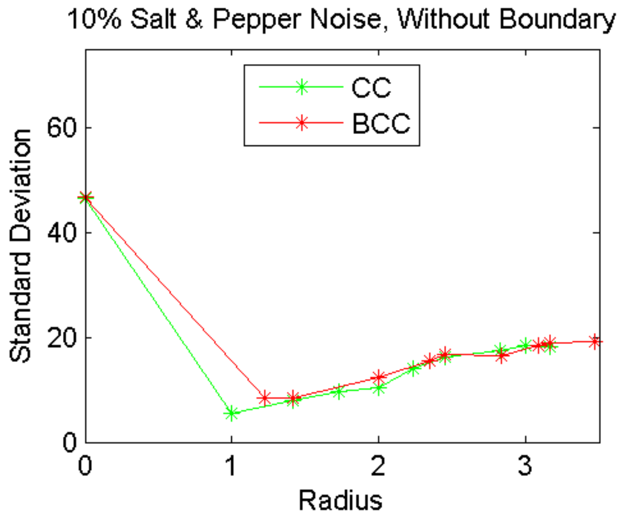


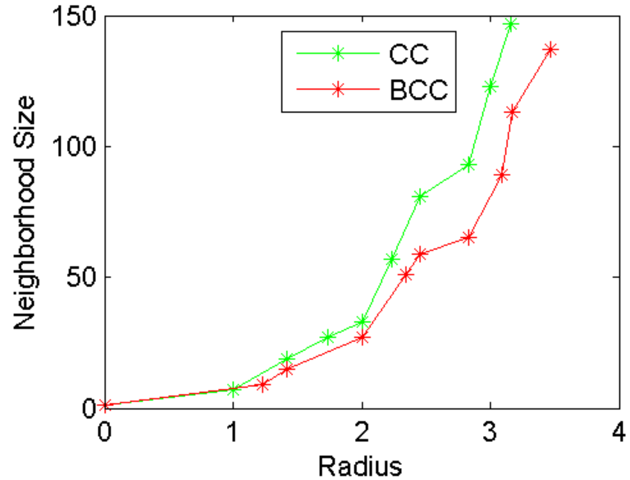


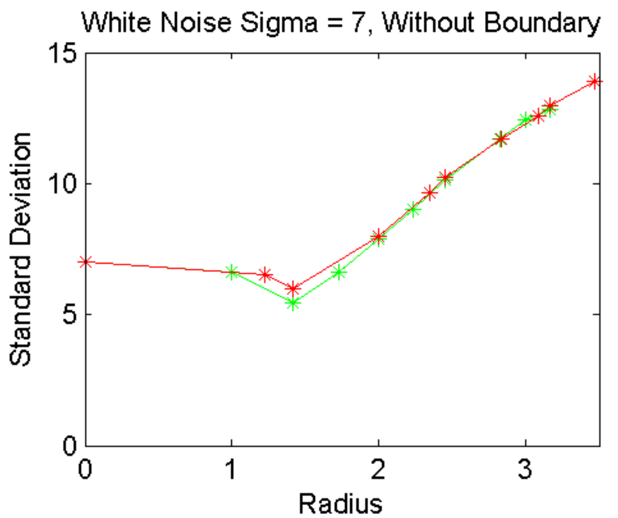


Index	CC Radius	CC Size	BCC Radius	BCC Size
1	0.000000	1	0.000000	1
2	1.000000	7	1.226058	9
3	1.414214	19	1.415730	15
4	1.732051	27	2.002145	27
5	2.000000	33	2.347723	51
6	2.236068	57	2.452117	59
7	2.449490	81	2.831461	65
8	2.828427	93	3.085513	89
9	3.000000	123	3.165669	113
10	3.162278	147	3.467817	137

Plot: Medium Noise







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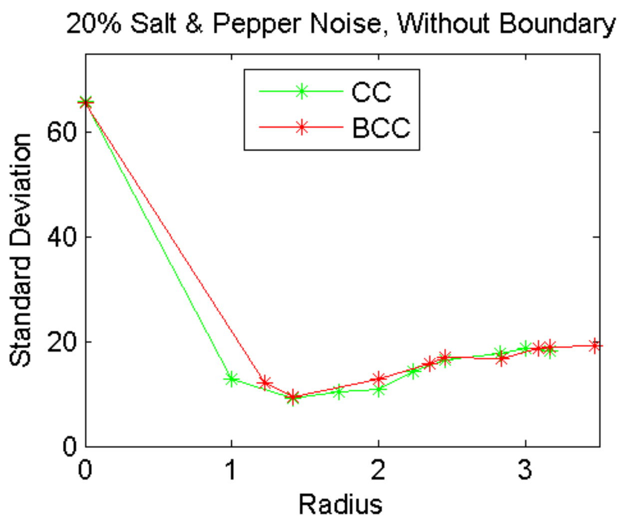
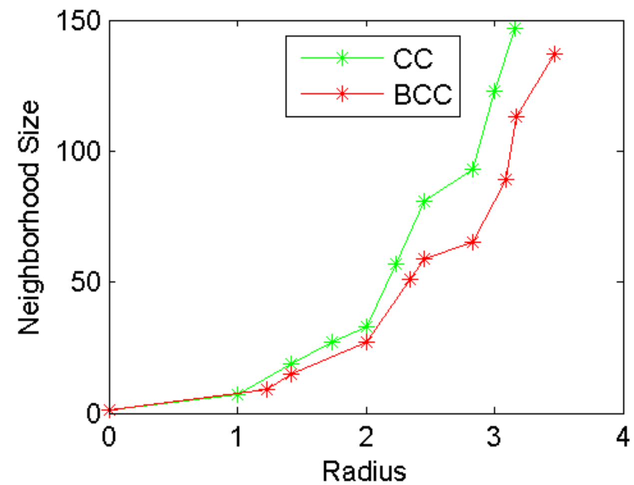
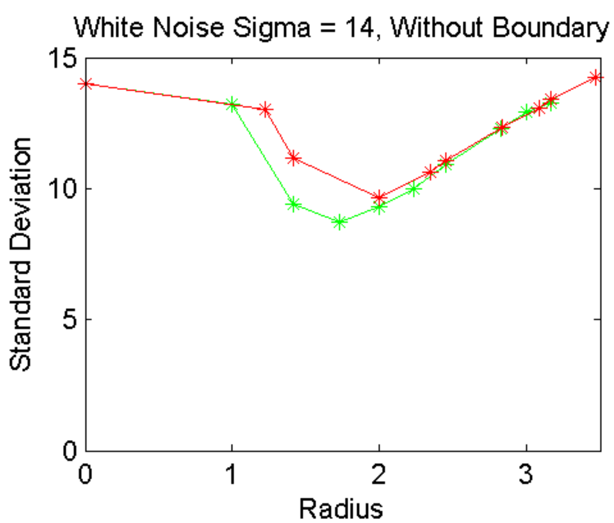
8	2.828427	93	3.085513	89
9	3.000000	123	3.165669	113
10	3.162278	147	3.467817	137

Plot: High Noise







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9	3.000000	123	3.165669	113
10	3.162278	147	3.467817	137

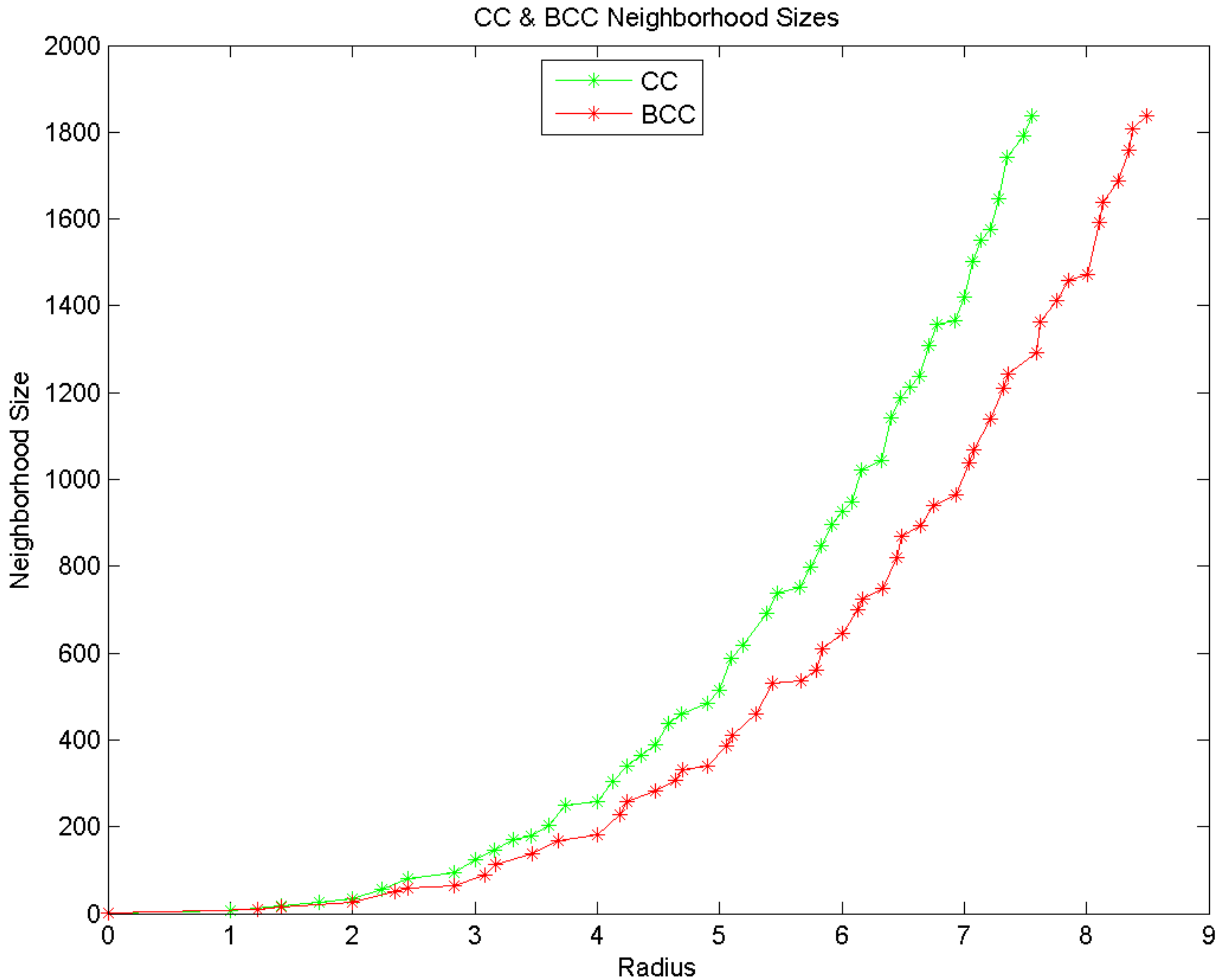
Conclusion

- Median filtering
  - BCC seems comparable to CC
- Gaussian filtering
  - BCC seems better for low noise levels
  - CC seems better for higher noise levels
  - Need further investigation

Bonus: Neighborhood Plot

- Investigate the claim that the ratio of BCC over CC neighbourhood sizes converge to roughly 0.7 (i.e. 1/sqrt(2))
- Plotted first 50 BCC/CC ratios
  - Can see convergent behaviour
  - Know that in the limit, ratio = 1/sqrt(2)

Bonus: Neighborhood Plot



References

- Robert Fisher, Simon Perkins, Ashley Walker and Erik Wolfart. Digital Filters. Department of Artificial Intelligence, University of Edinburgh, UK, 2003.
- Complete list:
  - http://www.taimeng.com/grad_school/CMPT775_proj/references.htm

Thank You!